Programmable Stiffness Regions on Metamaterials with Reconfigurable Behavior

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4. Methods

Oriedita:
(Origami crease pattern editor)

1. Built Miura Ori tessellation off 32 x 32 grid (256 Miura cells)
2. Designed inscription “defects” onto the Miura Ori which deliberately violated Maekawa's and Kawasaki's theorems

Custom Program:
(.stl -> .fold)

1. Parsed .fold file
2. Converted 2D vertices to 3D, triangulated faces, and built a crease parameter list (targets dihedral angles at crease **(2)**)
3. Mesh constructed from triangulated geometry; fold state maintained via crease torque formulation **(4)**
4. stl export captured current geometry, undoes internal scaling, and writes binary .stl

ANSYS:
(FE software)

1. Used facet clean up tools to allow precise discretization on imported .stl files
2. Realized using ANSYS wouldn’t be feasible because it is difficult to parametrically sweep fold angles (ANSYS was used sloley to set up mesh)

Custom Program:
(Force analysis)

1. Solver computed forces from deviation between current and target dihedral angles **(2), (4)**
2. Scaled to all target angles; solver continuously relaxed mesh to match
3. Stiffness scaled by crease type and length **(5)**; axial bar forces resisted edge stretching **(3)**
4. Nodal displacements solved by projecting forces onto rigid constraint null space **(6)**; solver continuously relaxed mesh to match target fold state

Custom Program:
(Result visualization)

1. Strain visualized as a continuous color gradient across the mesh (hue scaling)
2. Global average strain tracked across fold percentage sweep to quantify overall deformation
3. Visual strain gradients compared between “defect” and “non-defect” regions to identify structural impact of programmable stiffness regions
4. Made physical models to simulate the effects in practice

Equations

Face unit normal

$$n = \frac{(C-B)\times(A-B)}{||(C-B)\times(A-B)||} \quad (1)$$

Dihedral angle at crease

$$\theta = atan((n_1 \times \hat{e}) \cdot n_2, n_1 \cdot n_2) \quad (2)$$

Rigid bar axial force

$$f_{axial} = k_{axial}\frac{\Delta l}{L_0} \quad (3)$$

Crease angle error & torque

$$\Delta\theta = \theta - p\theta_{target}, \; M = K\Delta\theta \quad (4)$$

Crease stiffness

$$K = k_{crease}L(crease) \text{ or } k_{panel}L(face) \quad (5)$$

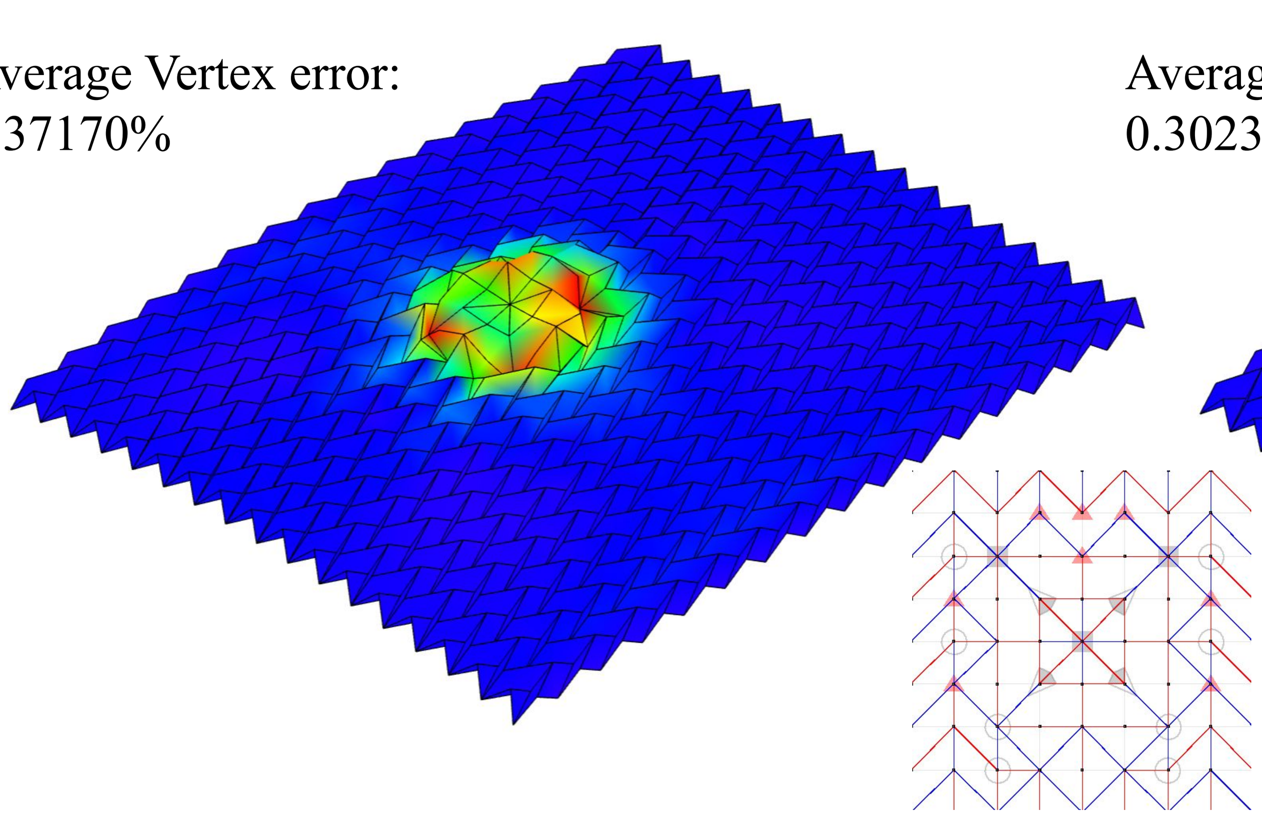
Constrained displacement solve

$$X = (I - (C^T)^+ C)F \quad (6)$$

5. Results

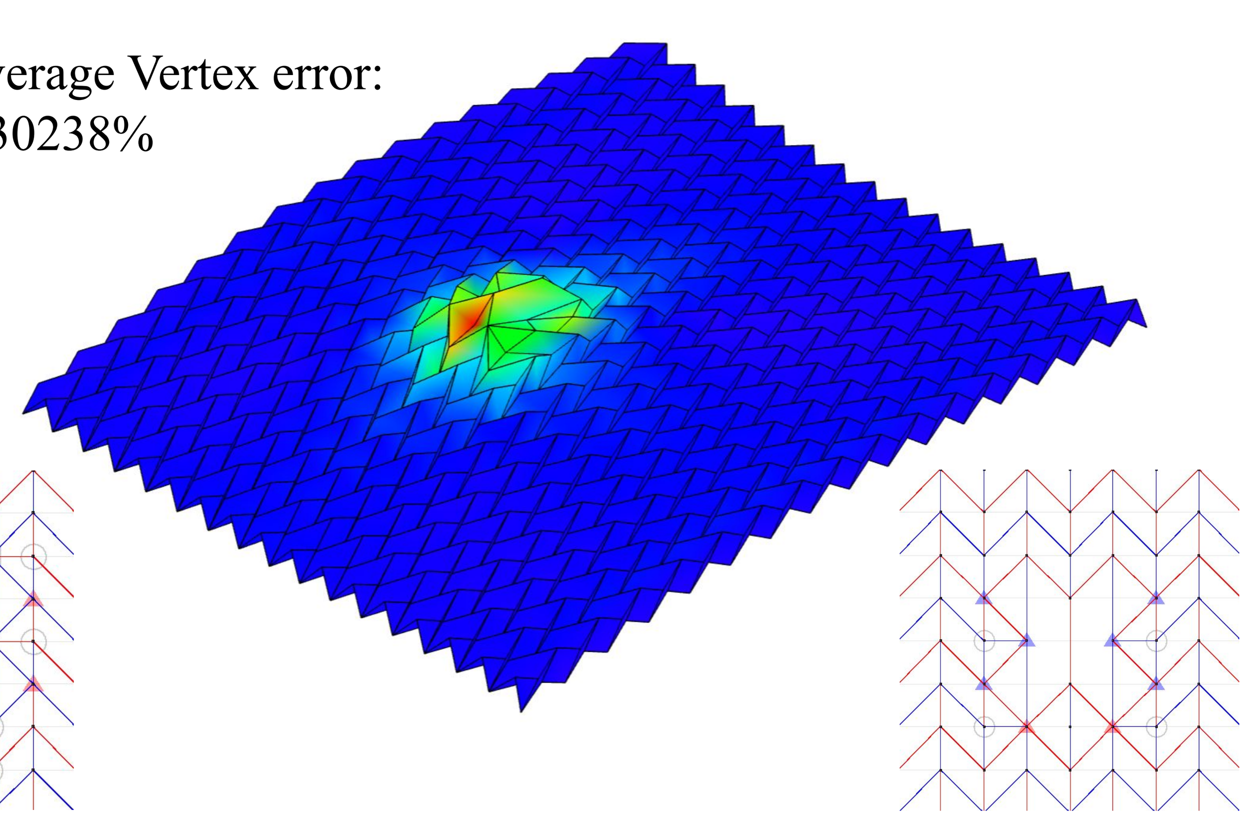
Box pleat (4x4):

Average Vertex error: 0.37170%



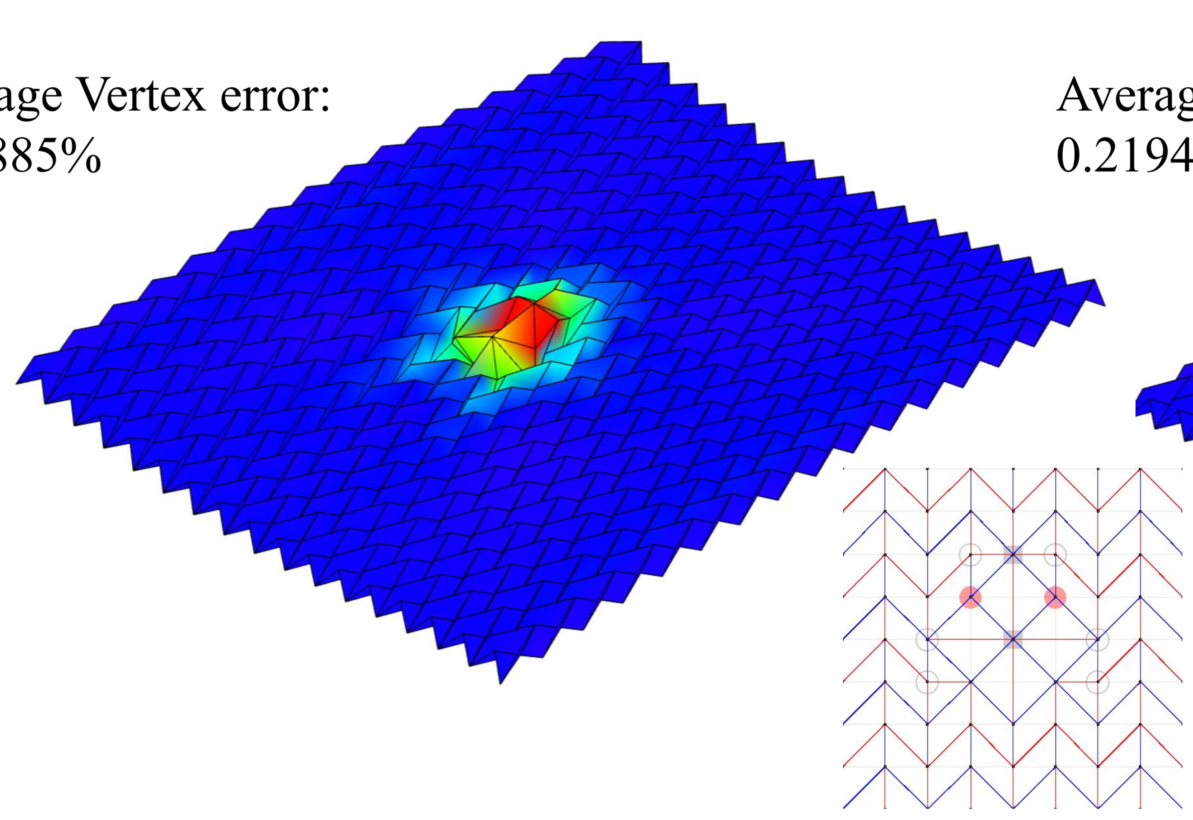
Box pleat (2x4):

Average Vertex error: 0.30238%



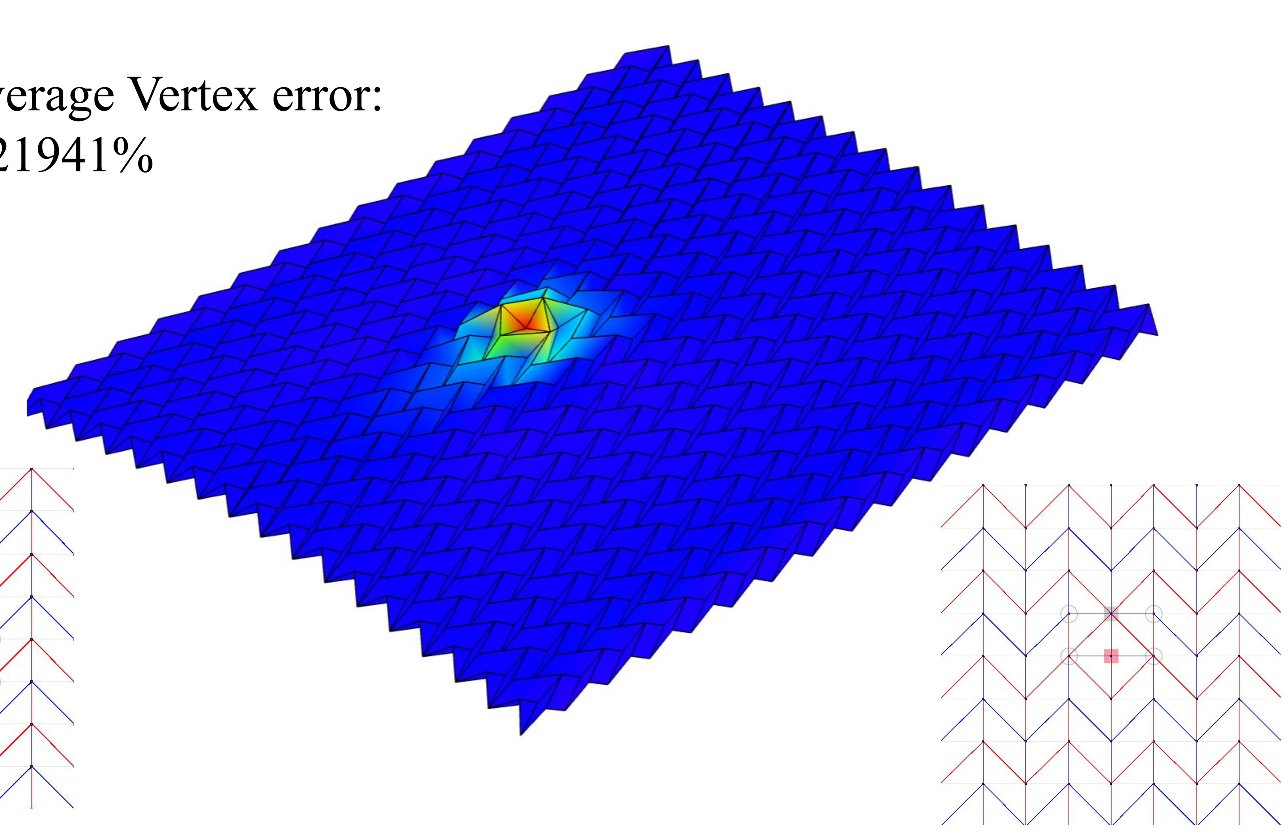
Water Bomb (full):

Average Vertex error: 0.26885%



Water Bomb (half):

Average Vertex error: 0.21941%



Strain Simulation Settings
Max strain: 5%
Folded state: 60%
Stiffness ratio (Axial : Face : Fold : Facet): 200 : 2 : 2 : 7 : 7
Damping ratio: 0.45
Folding speed: 10 seconds

Flat Foldability Violation Key

- 2 less mountain or valley fold
- 2 less mountain fold
- 2 less valley fold
- 1 less mountain fold
- 1 less valley fold
- Alternating sum of incident angles ≠ 0

Maximum Foldability
(Physical Experimentation)

- Box pleat (4x4): ~120°
- Box pleat (2x4): ~140°
- Water Bomb (full): ~125°
- Water Bomb (half): ~135°

Visualization tools used:
3D strain contour map — Custom Program
Crease Patterns — Oriedita

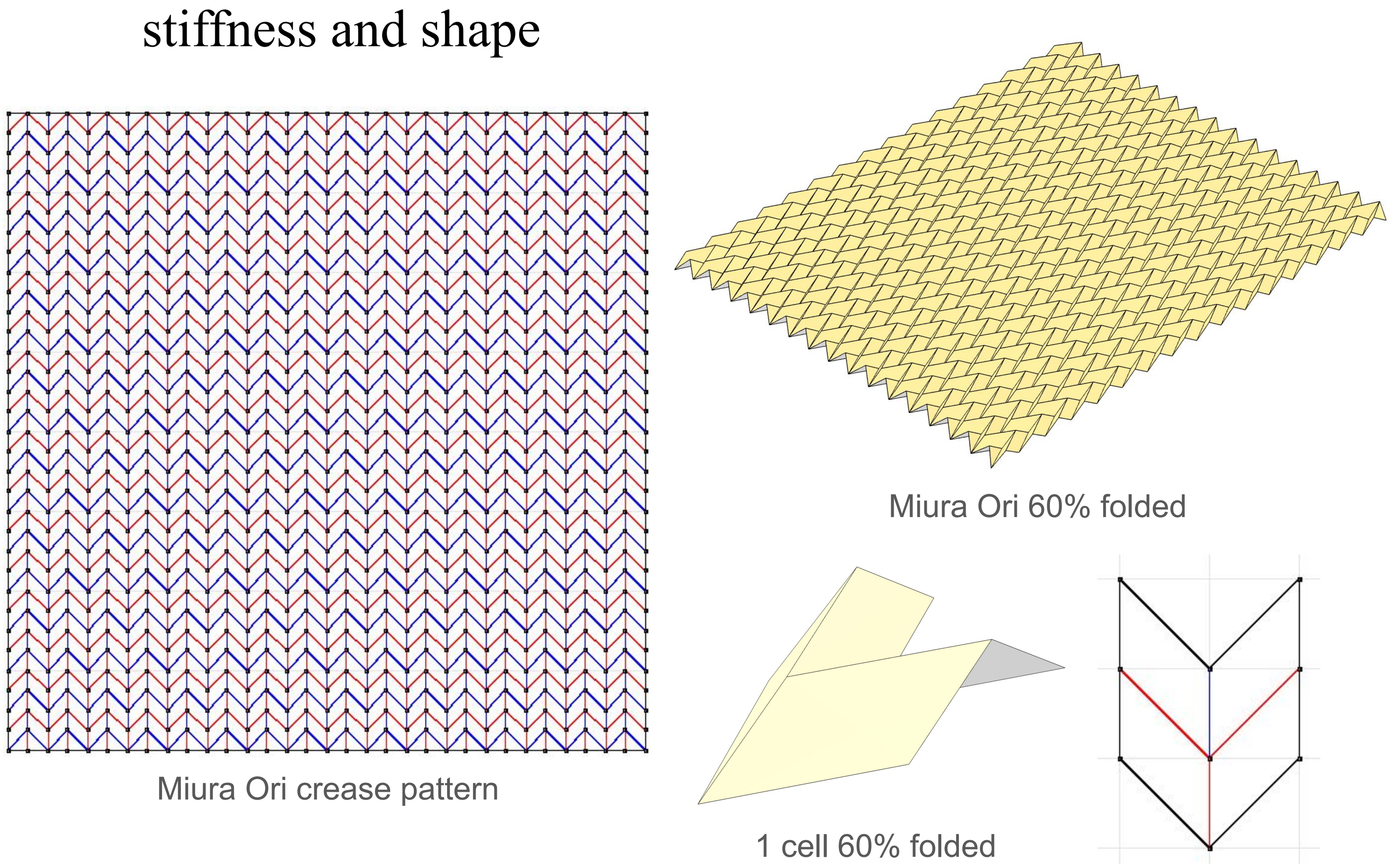
1. Background

Terminology:

- Miura-Ori — A periodic origami tessellation of alternating parallelograms that exhibits auxetic behavior
- Mechanical Metamaterial — Engineered structure whose mechanical properties arise from geometry
- Reconfigurable Structure — A system capable of switching between mechanical states
- Programmable Stiffness — Spatially controlled variation of rigidity through geometric modification

Related Research:

- (1) Jesse L. Silverberg, et al. (2014)
  - Introduced the concept of adding "pop-through" defects on the Miura-Ori
- (7) Schenk & Guest (2013)
  - Showed tunable mechanical properties from geometric parameters
- (8) Waitukaitis et al. (2015)
  - Built 3D origami-inspired metamaterials with tunable stiffness and shape



Miura Ori crease pattern

Miura Ori 60% folded

1 cell 60% folded

1 cell crease pattern

2. Objective

Motivation:

- Origami has potential in the field of engineering due to its' unique characteristics especially in applications such as deployable or reconfigurable structures.
- Specifically, the Miura-Ori tessellation has a negative Poisson's ratio, high structural integrity, great rigidity, and efficient compressibility.
- In order to effectively apply origami-inspired metamaterials in industries, it would help if we are able to control the characteristics to fit their needs precisely.

Goal:

- The goal is to make the Miura-Ori tessellation versatile for real-world applications by developing controlled modifications that "fine-tune" how it folds and unfolds.
- The controlled modifications include strategic "defects" that create regions of stiffness forcing the Miura-Ori to exhibit desired reconfigurable behaviors.

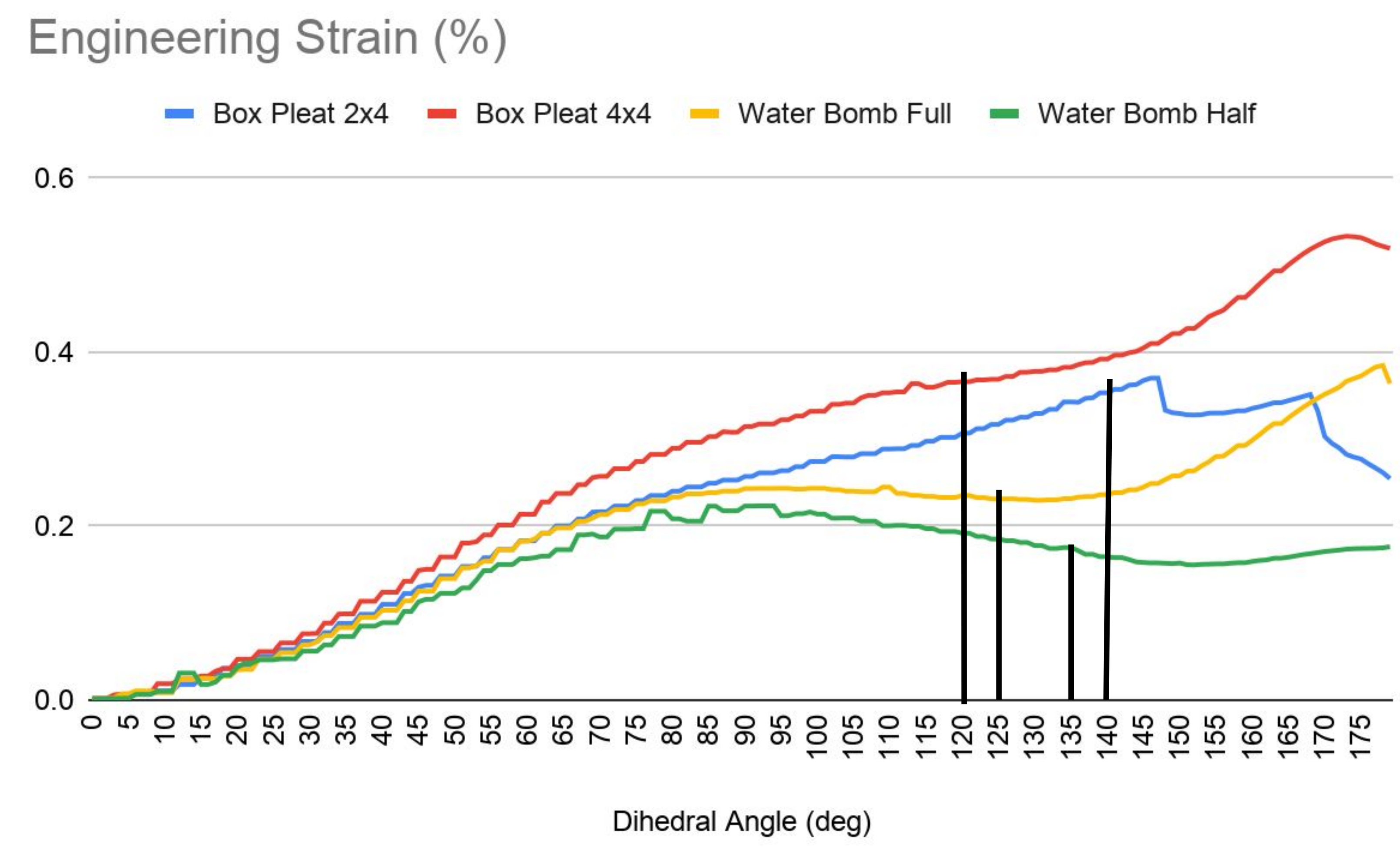
3. Hypothesis

Introducing strategically placed geometric defects will cause defect-induced regions to exhibit:

- Higher stress concentration
- Reduced flexibility
- Control over predictable folding patterns

6. Conclusion

Geometric defects could create programmable stiffness regions in the Miura-Ori tessellation.



"Defect" Properties				
	Box Pleat (4x4)	Box Pleat (2x4)	Water Bomb (full)	Water Bomb (half)
Peak Strain (Within foldable range)	~0.37%	~0.35%	~0.23%	~0.22%
Stiffness Behavior	Continuously stiffening	Stiffening → snap-through	Plateau → restiffening	Plateau → strain relief
Straining Pattern (Within foldable range)	Monotonic	Monotonic	Non-monotonic	Non-monotonic
Warping	Horizontal bending (backwards)	Horizontal bending (backwards) & Vertical bending (upwards)	Vertical bending (upwards)	Vertical bending (backwards)

7. Future Research

- Multiple "defects" on the same tessellation
- More experimentation with variety of material properties (stiffness)
- Different types of "defects"
  - Different dimensions of Box Pleat "defect"
  - Variations of pop-through "defects"
- Analyze different types of properties
  - Force distribution
  - Elasticity
  - Effects on Poisson's Ratio

8. References

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